TEST YOUR SKILL SOLUTIONS

Problem One

To find speed at start of the skid, skid distance, the friction and the basic skid formula. The variables:

S = Speed of the vehicle
d = Skid distance = 217 ft.
f = Drag factor = 0.51

Substituting into the formula:

$$S = \sqrt{30*d*f} = \sqrt{30*217*.51}$$

$$S = \sqrt{3320} = 57.62 \text{ or } 58 \text{ mph [93 kph]}$$

Problem Two

To calculate the average, we need to utilize the equation that gives the relation between acceleration, initial velocity and distance.

$$d = d_O + V_O*t + 0.5*a*t^2$$

Where:
- d = Total distance = 150 feet
- d_O = Starting point = 0 feet
- V_O = Initial velocity = 0 feet/sec
- a = acceleration rate
- t = time = 3.9 seconds

Substituting and solving:

$$150 = 0 + 0*3.9 + 0.5*a*3.9^2$$

$$150 = 0.5*a*15.21 = 7.605 * a$$

$$a = 150 / 7.6 = 19.72 \text{ ft/sec/sec [6.0 m/s/s]}$$

Velocity at the end of the test can be estimated by multiplying the acceleration by the time elapsed:

$$V = a * t = 19.72 * 3.9 = 76.9 \text{ ft/sec}$$

Converting to miles per hour:

$$S = V/1.467 = 76.9/1.467 = 52.4 \text{ or } 52 \text{ mph [84 kph]}$$

Problem Three

We have a conservation of momentum problem on our hands. The variables:

S_1 = Speed of V-1 at impact
s_1 = Speed of V-1 after impact
S_2 = Speed of V-2 at impact = 0 mph
s_2 = Speed of V-2 after impact
W_1 = Weight of V-1 = 3025 lb.
W_2 = Weight of V-2 = 2807 lb.
d_1 = Post impact skid distance of V-1 = 20 feet
d_2 = Post impact skid distance of V-2 = 43 feet
f_1 = Post impact drag factor of V-1 = 0.76
f_2 = Post impact drag factor of V-1 = 0.47

The weight ratios: If $W_1 = 1$, then $W_2 = 2807/3025 = 0.928$

Post impact speed of V-1:

$$s_1 = \sqrt{30*d_1*f_1} = \sqrt{30*20*.76} = 21.3 \text{ mph}$$

Post impact speed of V-2:

$$s_2 = \sqrt{30*d_2*f_2} = \sqrt{30*43*.47} = 24.6 \text{ mph}$$

The conservation of momentum formula:

$$S_1*W_1 + S_2*W_2 = s_1*W_1 + s_2*W_2$$

$$S_1*1 + 0*.928 = 21.3*1 + 24.6*.928$$

$$S_1 + 0 = 21.3 + 22.8$$

$$S_1 = 44.1 \text{ or } 44 \text{ mph [71 kph]}$$

Problem Four

The variables:
S = Speed of the car = 70 mph
d_R = Distance covered while driver reacts
d_S = Distance required to brake to stop
d = Total distance
f = Drag factor = 0.65
t = Reaction time = 1.15 sec

To find the total distance needed to react and stop, we add the distance covered during the 1.15 second reaction phase and the distance needed to brake to a stop. The former is determined using a variant of the time/distance/velocity relationship. The latter is determined using a variant of basic skid formula.

$$d = d_R + d_S = S * 1.467 * t + S^2 / 30*f$$

$$d = 70*1.467*1.15 + 70^2 / 30*.65$$

$$d = 118.0 + 251.3 = 369 \text{ feet [112 m]}$$

This means the car will stop 369 - 0 = 69 feet beyond the point of impact. Note that 1.15 seconds rather than the "standard" 1.6 seconds was chosen for evaluation of this scenario. It is the opinion of the editor that the stimulus provided by the large yellow vehicle and all of its warning lights would substantially reduce the perception and judgement phases of the reaction process.

For more on reaction time see *Accident Reconstruction Journal* Jan./Feb. 2001 pg. 58.

Problem Five

Solution of this problem has two main steps. We will ultimately utilize the critical speed to sideslip equation (banked surface). First we must determine the radius of curvature of the tire mark used to estimate the radius of curvature of the center of mass. The equation:

$$R = \frac{C^2}{8*m_O} + \frac{m_O}{2}$$

Where:
- R = Radius of curvature, feet
- C = Chord length = 60 feet
- m_O = Middle ordinate length, feet = 20.5" / 12 = 1.7 feet

Substituting and solving:

$$R = \frac{60^2}{8*1.7} + \frac{1.7}{2} = 266 \text{ ft.}$$

We now plug the radius of curvature, lateral friction coefficient and cross slope into the critical speed to sideslip formula. The equation:

$$S = \frac{\sqrt{15*R*(f+m)}}{\sqrt{(1 - f*m)}}$$

Where:
- S = Speed in miles per hour
- f = Lateral friction coefficient = .74
- R = Radius of curvature = 266 ft
- m = cross slope = 2% = 2/100 = 0.02

Substituting and solving:

$$S = \frac{\sqrt{15*266*(.74+.02)}}{\sqrt{(1 - .74*.02)}} = \frac{\sqrt{3032}}{\sqrt{.985}}$$

$$S = 55.06 / .993 = 55.45 \text{ or } 55 \text{ mph [89 kph]}$$

Problem Six

We will use the one-dimensional conservation of momentum formula to determine POST impact speeds of the vehicles. Knowing impact speeds we can then subtract one from the other to get speed change (ΔV). We'll call the car V-1 and the school bus V-2. The variables:

S_1 = Speed of V-1 at impact = 63 mph
S_2 = Speed of V-2 at impact = 0 mph
s = Speed of vehicles after impact
W_1 = Weight of V-1 = 3770 lb.
W_2 = Weight of V-2 = 17,650 lb.
ΔV_1 = Speed change of V-1
ΔV_2 = Speed change of V-2

The weight ratios: If $W_1 = 1$, then $W_2 = 17{,}650/3770 = 4.68$

The conservation of momentum formula:

$$S_1*W_1 + S_2*W_2 = s_1*W + s*W_2$$

$$63*1 + 0*4.68 = s*1 + s*4.68 = s*5.68$$

$$s = 63 / 5.68 = 11.1 \text{ mph}$$

We will subtract post impact speeds from impact speed to get speed change (ΔV) for each vehicle.

$\Delta V_1 = s - S_1 = 11.1 - 63 = -51.9$ mph (speed loss) [84 kph]

$\Delta V_2 = s - S_2 = 11.1 - 0 = 11.1$ mph (speed gain) [18 kph]

Problem Seven

To solve this problem we need the following information: the engine crankshaft speed in revolutions per minute, RPM, the transmission (gear) ratio, the number of times the engine crankshaft will turn for every time the driveshaft turns, i_T, the axle ratio, the number of times the driveshaft will turn for every time the axle shaft turns, i_A, and the radius of the drive wheels. Given that there are two times pi (3.1416) inches of circumference for every inch of a tire's radius, times 60 minutes in every hour, divided by 12 inches in every foot and divided by 5280 feet in every mile, leaves us with a conversion constant of 0.00595. Therefore our equation is:

$$S = \frac{0.00595 * RPM * R}{i_T * i_A}$$

Where:
S = Speed of vehicle, in mph
RPM = Engine crankshaft speed = 700 or 2200 rev./minute
R = Radius of the tire = 26 inches
i_T = Gear ratio = 2.57
i_A = Axle ratio = 3.44

To get the lowest possible impact speed we need to use the above equation assuming the lowest possible engine speed (700 rpm):

$$S = \frac{0.00595 * 700 * 26}{2.57 * 3.44} = 12.2 \text{ or } 12 \text{ mph [20 kph]}$$

To get the highest impact speed we use the highest engine speed (2200 rpm):

$$S = \frac{0.00595 * 2200 * 26}{2.57 * 3.44} = 38.49 \text{ or } 38 \text{ mph [62 kph]}$$

Problem Eight

The Campbell model, in equation form:

$$ebs = b_1 * C_{AVE} + b_0$$

Where:
ebs = Equivalent barrier speed, mph = 29.6 mph
b_1 = Stiffness constant, mph/in
C_{AVE} = Average crush depth, in = 17.3"
b_0 = Damage threshold constant, given 7 mph in this problem

The equation can also be generated for metric units. The b_1 and b_0 would have to be different than the imperial unit equation.

Assume b_0 = 7 mph

$$ebs = b_1 * C_{AVE} + b_0$$
$$29.6 = b_1 * 17.3 + 7$$
$$22.6 = b_1 * 17.3$$
$$1.31 = b_1$$

Therefore, the equation: $ebs = 1.31*C_{AVE} + 7$

The CRASH3 stiffness coefficients can also be derived for the Volvo. We will use curb weight (W = 3205 lb), the average crush, a 7 mph damage threshold, and the damage width (L = 63.9 in).

Since we are using 7 mph as the damage threshold constant, the b_1 and b_0 constants will be the same, except that mph must be converted to inches per second:

$$b_0 = 7 \text{ mph} * \frac{1.467 \text{ ft/sec}}{\text{mph}} * \frac{12 \text{ in}}{\text{ft}} = 123.2 \text{ in/sec}$$

$$b_1 = \frac{1.31 \text{ mph}}{\text{in}} * \frac{1.467 \text{ ft/sec}}{\text{mph}} * \frac{12 \text{ in}}{\text{ft}} = 23.1 \text{ /sec}$$

The CRASH3 constants:

$$A = \frac{W*b_0*b_1}{g * L} = \frac{3205*123.2*23.1}{386.4 * 63.9} = 369$$

$$B = \frac{W*b_1*b_1}{g * L} = \frac{3205*23.1*23.1}{386.4 * 63.9} = 69$$

$$G = A^2 / 2*B = 369^2 / 2*69 = 986$$

Note that 'g', the acceleration of gravity, is given as (32.2*12) inches per second per second in order to make the units consistent.

Problem Nine

We have a conservation of momentum problem on our hands. The variables:

S_1 = Speed of V-1 at impact
s_1 = Speed of V-1 after impact
S_2 = Speed of V-2 at impact
s_2 = Speed of V-2 after impact
W_1 = Weight of V-1 = 3072 + 220 + 40 = 3332 lb.
W_2 = Weight of V-2 = 3719 + 160 + 70 + 100 = 4049 lb.
d_1 = Post impact travel distance of V-1 = 33 feet
d_2 = Post impact travel distance of V-2 = 45 feet
f_1 = Post impact drag factor of V-1 = 0.60
f_2 = Post impact drag factor of V-2 on guardrail = 0.45
α_1 = Travel angle of vehicle one at impact
α_2 = Travel angle of vehicle two at impact
β_1 = Travel angle of vehicle one after impact
β_2 = Travel angle of vehicle two after impact

We will start setting by assuming that South is our 0 degree line. All angles measured counter clockwise will be considered positive. We will tabulate our angles and then determine the trigonometric values:

	Direction	θ	sine θ	cos θ
α_1:	East	90°	1	0
α_2:	South	0°	0	1
β_1:	S 24° E	24°	.407	.914
β_2:	E 18° S	18°	.309	.951

The weight ratios: If $W_1 = 1$, $W_2 = 4049/3332 = 1.215$

Post impact speed of V-1:

$$s_1 = \sqrt{30*d_1*f_1} = \sqrt{30*33*.60} = 24.4 \text{ mph}$$

Post impact speed of V-2:

$$s_2 = \sqrt{30*d_2*f_2} = \sqrt{30*45*.45} = 24.6 \text{ mph}$$

The conservation of momentum formula along east-west axis:

$$S_1*W_1*\sin\alpha_1 + S_2*W_2*\sin\alpha_2 = s_1*W_1*\sin\beta_1 + s_2*W_2*\sin\beta_2$$

$$S_1*1*1 + S_2*1.215*0 = 24.4*1*.407 + 24.6*1.215*.309$$

$$S_1 = 9.93 + 9.23$$

$$\mathbf{S_1 = 19.16 \text{ or } 19 \text{ mph [31 kph]}}$$

The conservation of momentum along the north-south axis:

$$S_1*W_1*\cos\alpha_1 + S_2*W_2*\cos\alpha_2 = s_1*W_1*\cos\beta_1 + s_2*W_2*\cos\beta_2$$

$$S_1*1*0 + S_2*1.215*1 = 24.4*1*.914 + 24.6*1.215*.951$$

$$1.215 * S_2 = 22.30 + 28.42 = 50.72$$

$$\mathbf{S_2 = 41.75 \text{ or } 42 \text{ mph [67 kph]}}$$

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